

# Heavy quark production at a $\gamma\gamma$ collider: the effect of large logarithmic perturbative corrections

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**Abstract.** We present quantitative results on the cross section for  $\gamma + \gamma (J_z = 0) \rightarrow b\bar{b} \rightarrow$  two  $b$ -jets based on the recently achieved all orders resummation of large soft (Sudakov) and hard (non-Sudakov) double logarithms. The next-to-leading order QCD perturbative corrections are included exactly. We find that one needs to include at least four loops for the novel hard leading logarithms, on the cross section level, in order to be safe from large opposite sign cancellations that plagued earlier phenomenological studies. We find that the background to intermediate mass Higgs boson production at a future photon linear collider (PLC) is thus reasonably well under control and should allow the direct determination of the partial Higgs width  $\Gamma(H \rightarrow \gamma + \gamma)$ . Assuming high efficiency  $b$ -tagging, the total Higgs width measurement at a PLC is thus a realistic goal.

## 1 Introduction

The measurement of the total Higgs width is one of the most important goals of a future  $\gamma\gamma$  collider. Besides providing various other physics opportunities, such as the determination of the CP eigenvalues of a Higgs boson, the photon linear collider (PLC) offers so far the only possibility of a direct measurement of the partial width  $\Gamma(H \rightarrow \gamma + \gamma)$  for an intermediate mass Higgs particle [1]. With the knowledge of the respective branching ratio  $\text{BR}(H \rightarrow \gamma + \gamma)$  from a Higgs production process at some earlier collider experiment [2,3], the total Higgs width can be reconstructed. This in turn permits a model independent determination of various other partial widths.

Using Compton backscattering [4,5] of initially polarized electrons and positrons at a linear collider, the dominant background for a Higgs boson below the  $W^\pm$  threshold is from  $\gamma + \gamma (J_z = 0) \rightarrow b + \bar{b}$ . While this background is suppressed by  $\frac{m_b^2}{s}$  at the Born level (unlike the  $J_z = \pm 2$  background which has no such mass suppression), higher-order perturbative QCD corrections remove this suppression [6]. In addition, very large virtual (non-Sudakov) double logarithms (i.e.  $\log^2(s/m_b^2)$ ) are present at each order in perturbation theory, and at one loop these can lead to a *negative* cross section in the central production region where the Higgs signal is expected [7]. In [9] it was shown that positivity of the cross section is restored by the leading two-loop non-Sudakov logarithmic corrections

and recently, in [10], these corrections were resummed to all orders.

The problem with previous [6,7] phenomenological studies was that the large higher-order leading logarithmic corrections were not known and therefore not included. It is the purpose of this study to quantify the effect of these higher-order logarithms on the cross section. We include all the results presently available — the exact one-loop correction [7] and the leading double logarithms to all orders — to make more reliable predictions for the Higgs background.

We do not attempt here a full Monte Carlo study of the signal and background taking all hadronization and detector effects into account. Instead, we note that the dominant configuration of the Higgs signal process is a pair of back-to-back jets, produced centrally, each containing a  $b$  quark that can in principle be tagged using a vertex detector. We therefore calculate the background cross section for the same topology. Note that this means excluding multi( $\geq 3$ )-jet topologies and also two-jet configurations arising from Compton-like  $2 \rightarrow 3$  scatterings, as studied in detail in [6], where only one of the two jets contains a  $b$  quark.

For simplicity, we normalize all our cross sections to that of the leading order  $\gamma + \gamma (J_z = 0) \rightarrow b + \bar{b}$  process. The Born amplitude for this is given by

$$\mathcal{M}_{\text{Born}}(\lambda_\gamma, \lambda_q) = \frac{8\pi\alpha Q_q^2}{(1 - \beta^2 \cos^2 \theta)} \frac{2m_q}{\sqrt{s}} (\lambda_\gamma + \lambda_q \beta) \quad (1)$$

where  $\lambda_\gamma$  and  $\lambda_q$  label the helicities of the photon and quark respectively. The Born cross section for the ( $J_z = 0$ )

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helicity state is then

$$\frac{d\sigma_{\text{Born}}}{d\cos\theta}(\gamma + \gamma \longrightarrow q + \bar{q}) = \frac{12\pi\alpha^2 Q_q^4}{s} \frac{\beta(1 - \beta^4)}{(1 - \beta^2 \cos^2\theta)^2} \quad (2)$$

where  $\beta = \sqrt{1 - \frac{4m_q^2}{s}}$  denotes the quark velocity. Here  $Q_q$  is the charge of the quark with mass  $m_q$ ,  $\alpha = \frac{1}{137}$  is the fine structure constant,  $\theta$  is the quark scattering angle in the center-of-mass frame, and  $\sqrt{s}$  is the overall center-of-mass scattering energy.

The paper is structured as follows. We begin in the next section by summarizing the results of [10] and discussing the real gluon emission contributions. Numerical results are given in Sect. 3 and we make concluding remarks in Sect. 4.

## 2 Higher-order corrections

The Born cross section for polarized  $\gamma\gamma$  ( $J_z = 0$ )  $\rightarrow q\bar{q}$  collisions given in (1) receives large  $\sim (\alpha_s \log^2(s/m^2))^n$  corrections at each order in perturbation theory. These ‘novel’ hard<sup>1</sup> double logarithms arise from corrections which are effectively cut off by *quarks*, rather than soft gluons which generate the usual Sudakov double logarithms. In [10] we showed how these new double logarithmic (DL) corrections can be resummed to all orders in perturbation theory. The result takes the form of a confluent hypergeometric function  ${}_2F_2$  which possesses a  $\log\left(\frac{\alpha_s}{\pi} \log^2 \frac{m^2}{s}\right)$  high energy limit. Taking into account the  $\frac{m}{\sqrt{s}}$  suppression contained in the Born amplitude (1), the new corrections are well behaved as  $s \rightarrow \infty$ . The series expansion agrees with the known one- and two-loop results of [7, 9] as well as our explicit three-loop calculation [10].

It was furthermore shown in [10] that at three loops all additional doubly logarithmic corrections are described by the exponentiation of Sudakov logarithms at each order in the expansion of the new hard form factor. This behavior is already present at one and two loops and can thus safely be extrapolated to all orders.

For completeness, we list here the resulting virtual DL-form factor contribution to the amplitude for the process  $\gamma + \gamma$  ( $J_z = 0$ )  $\rightarrow q + \bar{q}$ :

$$\begin{aligned} \mathcal{M}_{\text{DL}} &= \mathcal{M}_{\text{Born}} \left\{ \exp(\mathcal{F}_{\mathcal{A}}) + \mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{1}{2}\mathcal{F}\right) \right. \\ &\quad + 2\mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{C_A}{4C_F}\mathcal{F}\right) \\ &\quad + \mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{1}{2}\mathcal{F}\right) [\exp(\mathcal{F}_{\mathcal{A}}) - 1] \\ &\quad \left. + 2\mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{C_A}{4C_F}\mathcal{F}\right) [\exp(\mathcal{F}_{\mathcal{A}}) - 1] \right\} \\ &= \mathcal{M}_{\text{Born}} \left\{ 1 + \mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{1}{2}\mathcal{F}\right) \right. \end{aligned}$$

<sup>1</sup> We use the description ‘hard’ to indicate that it is the heavy quark mass which acts as the effective infrared cutoff for the new double logarithms.

$$\left. + 2\mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{C_A}{4C_F}\mathcal{F}\right) \right\} \exp(\mathcal{F}_{\mathcal{A}}) \quad (3)$$

where

$$\mathcal{F}_{\mathcal{A}} \equiv -C_F \frac{\alpha_s}{2\pi} \left( \frac{1}{2} \log^2 \frac{m^2}{s} + \log \frac{m^2}{s} \log \frac{\lambda^2}{m^2} \right) \quad (4)$$

$$\mathcal{F} \equiv -C_F \frac{\alpha_s}{4\pi} \log^2 \frac{m^2}{s} \quad (5)$$

denote the soft and hard one-loop form factors, respectively. A fictitious gluon mass  $\lambda$  is introduced to regulate the infrared divergences in the former. Of course in a physical cross section, the soft form factor  $\mathcal{F}_{\mathcal{A}}$  cancels the corresponding infrared divergent contributions from the emission of real soft gluons. For the two-jet-like contributions considered in this work, i.e. soft gluon radiation below an energy cut of  $k_g \leq \epsilon\sqrt{s}$  and arbitrarily hard gluons collinear with one of the b-quarks (specified by a cone of half-angle  $\delta$ ), we need to make sure that the jet definition does not restrict the exponentiation of the energy cut dependent piece of the soft gluon matrix elements. In this case we are able to give the contribution of the real soft DL corrections to all orders by writing

$$\frac{d\sigma_{\text{virt+soft}}^{DL}}{d\cos\theta} \sim |\mathcal{M}_{\text{DL}}|^2 \exp\{-2\mathcal{F}_{\mathcal{A}} + \Delta_c\} \quad (6)$$

where the last term in the exponential  $\Delta_c$  depends on the soft-gluon cut prescriptions. Restricting the gluon energies by  $E_g \leq k_c$ , for example<sup>2</sup>, gives to leading logarithmic order,

$$\Delta_c = -\frac{\alpha_s C_F}{\pi} \log \frac{s}{m_q^2} \log \frac{s}{4k_c^2} \quad (7)$$

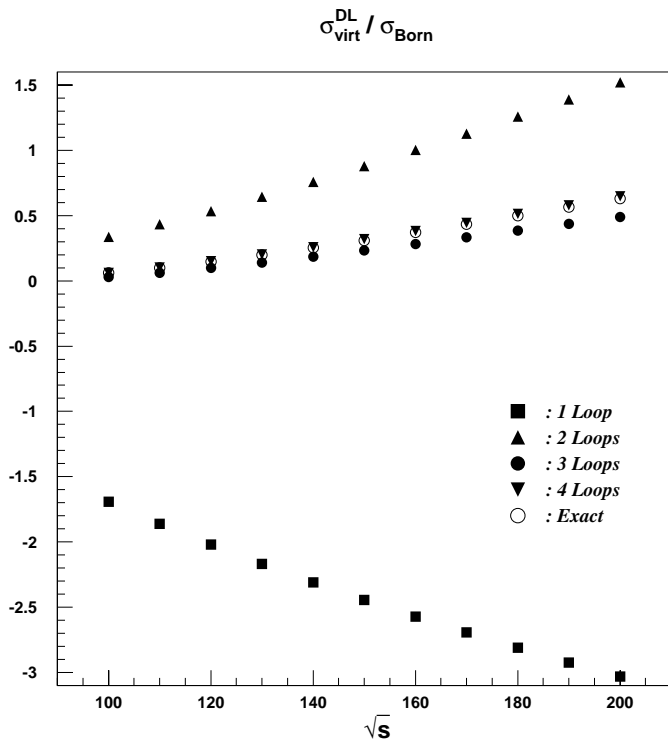
At one loop, however, such DL expressions are insufficient as the integrated real gluon contributions also include *sub-leading* logarithms with a cut dependence. For our jet definition, we are actually able to employ exact expressions for the infrared divergent functions as these have the same structure as in the QED case [11, 12]. We therefore use (assuming only  $m_q^2/s \ll 1$ ):

$$\Delta_c = -\frac{\alpha_s C_F}{\pi} \left( \log \frac{s}{m_q^2} \log \frac{s}{4k_c^2} - \log \frac{s}{4k_c^2} + \frac{\pi^2}{3} \right) \quad (8)$$

(cf. (7)), which leads to the physical cross section for virtual and soft ( $E_g < k_c$ ) real gluon emissions:

$$\begin{aligned} \sigma_{\text{virt+soft}}^{DL} &= \sigma_{\text{Born}} \left\{ 1 + \mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{1}{2}\mathcal{F}\right) \right. \\ &\quad \left. + 2\mathcal{F} \, {}_2F_2\left(1, 1; 2, \frac{3}{2}; \frac{C_A}{4C_F}\mathcal{F}\right) \right\}^2 \\ &\quad \exp\left( \frac{\alpha_s C_F}{\pi} \left[ \log \frac{s}{m_q^2} \left( \frac{1}{2} - \log \frac{s}{4k_c^2} \right) \right. \right. \\ &\quad \left. \left. + \log \frac{s}{4k_c^2} - 1 + \frac{\pi^2}{3} \right] \right) \quad (9) \end{aligned}$$

<sup>2</sup> One could also use an invariant mass (‘ $y_{\text{cut}}$ ’) type cutoff.

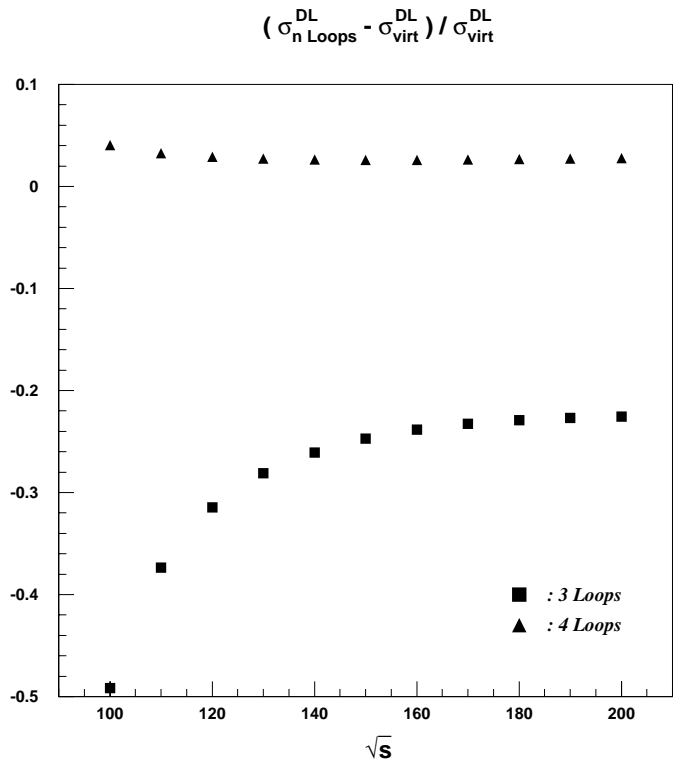


**Fig. 1.** The size of the virtual double logarithmic (DL) contributions relative to the Born cross section through four loops. The ‘exact’ result (open circles) is given by the all orders resummation according to (9) and is in very good agreement with the four-loop approximation given in (10). The huge one and two loop contributions can be seen to lead to physically distorted results

When combined with hard ( $E_g > k_c$ ) real gluon emission appropriate to some particular (i.e. two-jet-like) final state topology, the logarithmic dependence on the cutoff  $k_c$  will cancel and the Sudakov form factor will become  $\mathcal{O}(1)$ . The remaining double logarithmic corrections then come entirely from the hard  $\{\}^2$  non-Sudakov form factor. In order to demonstrate the numerical impact of these logarithms we list the expansion of the  $\{\}^2$  piece in (3). Using the coefficients in the series expansion for the hypergeometric function, we obtain the expansion through four loops:

$$\begin{aligned} \frac{\sigma_{\text{virt}}^{\text{DL}}}{\sigma_{\text{Born}}} &\sim 1 + 6\mathcal{F} + \frac{1}{6} \left( 56 + 2\frac{C_A}{C_F} \right) \mathcal{F}^2 \\ &+ \frac{1}{90} \left( 94 + 90\frac{C_A}{C_F} + 2\frac{C_A^2}{C_F^2} \right) \mathcal{F}^3 \\ &+ \frac{1}{2520} \left( 418 + 140\frac{C_A}{C_F} + 238\frac{C_A^2}{C_F^2} + 3\frac{C_A^3}{C_F^3} \right) \mathcal{F}^4 \\ &+ \mathcal{O}(\mathcal{F}^5) \end{aligned} \quad (10)$$

where  $\mathcal{F} = -C_F \frac{\alpha_s}{4\pi} \log^2 \frac{m^2}{s}$  is again the one-loop hard form factor. This expansion demonstrates that it is not sufficient, as suggested in [9], to only consider the first



**Fig. 2.** The relative size of the virtual double logarithmic contributions through three (squares) and four (triangles) loops. With the expected experimental precision [6] one needs to include at least four-loop corrections or simply the resummed values according to (9)

two coefficients in (21) of that paper (corresponding to the terms up to  $\mathcal{O}(\mathcal{F}^2)$  in the above expansion). While the amplitude form factor series in (3) converges well after the first two terms, the interference terms on the cross section level require the inclusion of the four loop contribution according to (10).

Figure 1 shows the respective contributions<sup>3</sup> of the terms listed in (10) relative to the Born cross section (2). For illustration, we use parameter values of  $\alpha_s = 0.11$  and  $m \equiv m_b = 4.5$  GeV, so that  $\mathcal{F}$  varies between  $-0.45$  at  $\sqrt{s} = 100$  GeV and  $-0.67$  at  $\sqrt{s} = 200$  GeV. The large cancellations between the lower order terms are clearly visible and only the four-loop expansion is close to the exact resummed result. The relative size of the doubly logarithmic three- and four-loop corrections with respect to the full answer are depicted in Fig. 2. While taking only the three-loop expansion leads to a deviation of up to 50% compared to the exact result, the four-loop DL cross section (10) stays within a few percent. In the next section we present the results based on inclusion of the full one-loop radiative corrections including the full two-jet-like bremsstrahlung contribution.

<sup>3</sup> Note that here the ‘ $n$ -loop contribution’ means the sum of the contributions up to and including the  $n^{\text{th}}$  order contribution in (10).

### 3 Numerical results

The results of the previous section confirm that the hard double logarithms *are* numerically important when one is considering  $b\bar{b}$  production in the  $\sqrt{s} = 100 - 200$  GeV energy range. The one-loop contribution on its own tends to drive the cross section negative, and stability is only achieved at four-loop order in the cross section. However before drawing definite conclusions, it is important to assess the other contributions to the physical cross section, in particular (i) the sub-leading logarithmic corrections to the virtual + soft form factors and (ii) the contributions from hard gluon emission. Unfortunately at present we know nothing about these beyond first order. However the first-order result *is* worth studying in detail, since it allows us to assess the dominance or otherwise of the ‘6 $\mathcal{F}$ ’ hard double logarithm term in (10).

We first need to define an infrared safe two-jet cross section.<sup>4</sup> As discussed in the Introduction, our interest here is in two jet like configurations, where each jet contains a  $b$  quark. It is convenient, for purposes of illustration, to use a modification of the Sterman–Weinberg (SW) two jet definition [13]. At leading order (i.e.  $\gamma\gamma \rightarrow b\bar{b}$ ) all events obviously satisfy the two- $b$ -jet requirement. We apply an angular cut of  $|\cos\theta_{b,\bar{b}}| < 0.7$  to ensure that both jets lie in the central region. This defines our ‘leading order’ (LO) cross section. At next-to-leading order (NLO) we can have virtual or real gluon emission. For the latter, an event is defined as two- $b$ -jet like if the emitted gluon

- either I. has energy less than  $\epsilon\sqrt{s}$ , with  $\epsilon \ll 1$ ,
- or II. is within an angle  $2\delta$  of the  $b$  or  $\bar{b}$ ,  
again with  $\delta \ll 1$ .

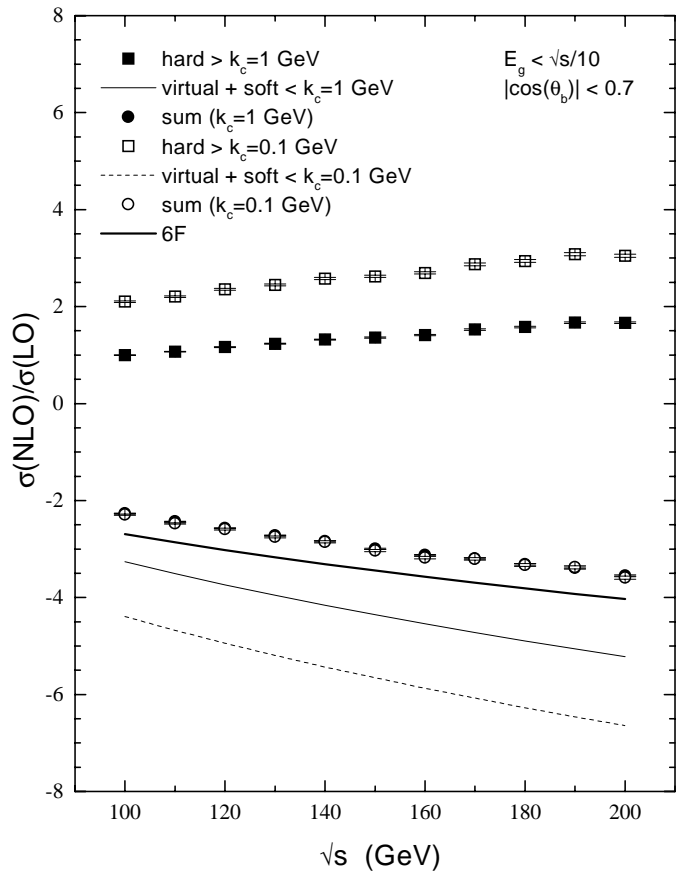
We will call the two regions of phase space I and II respectively. We further subdivide region I according to whether the gluon energy is greater or less than the infrared cut-off<sup>5</sup>  $k_c$  ( $< \epsilon$ ). We combine the region of soft real gluon phase space  $E_g < k_c$  with the virtual gluon contribution to give  $\sigma_{SV}$ . The region  $\bar{I}$  of ‘hard’ real gluon phase space  $k_c < E_g < \epsilon\sqrt{s}$  then defines  $\sigma_{\bar{I}}$ . The total NLO two- $b$ -jet cross section is then

$$\sigma_{\text{NLO}} = \sigma_{\text{SV}} + \sigma_{\bar{I}} + \sigma_{\text{II}} \quad (11)$$

The cross section for  $\sigma_{\text{SV}}$  is obtained from the analytic expression given in [7], while  $\sigma_{\bar{I}}$  and  $\sigma_{\text{II}}$  are computed using the numerical program of [6]. A powerful consistency check on the overall calculation is that the sum  $\sigma_{\text{SV}} + \sigma_{\bar{I}}$  should be independent of the unphysical parameter  $k_c$ . This is demonstrated in Fig. 3, which shows the individual contributions as a function of  $\sqrt{s}$  for two choices,  $k_c = 1$  GeV and 0.1 GeV and with  $\epsilon = 0.1$ . While the individual contributions are of course different (the leading behavior as  $k_c \rightarrow 0$  is  $\sim \pm \log(s/m_q^2) \log(s/k_c^2)$  for the separate

<sup>4</sup> Note that all cross sections discussed and calculated here correspond to the  $J_Z = 0$   $\gamma\gamma$  polarization state.

<sup>5</sup> Note that unlike  $\epsilon$ ,  $k_c$  is an unphysical parameter introduced simply to separate soft and hard real emission.



**Fig. 3.** Virtual and real ( $E_g < k_c, k_c < E_g < 0.1\sqrt{s}$ ) contributions to the exact next-to-leading order two- $b$ -jet cross section defined in the text. The hard double logarithm contribution at this order (6 $\mathcal{F}$ ) is also shown

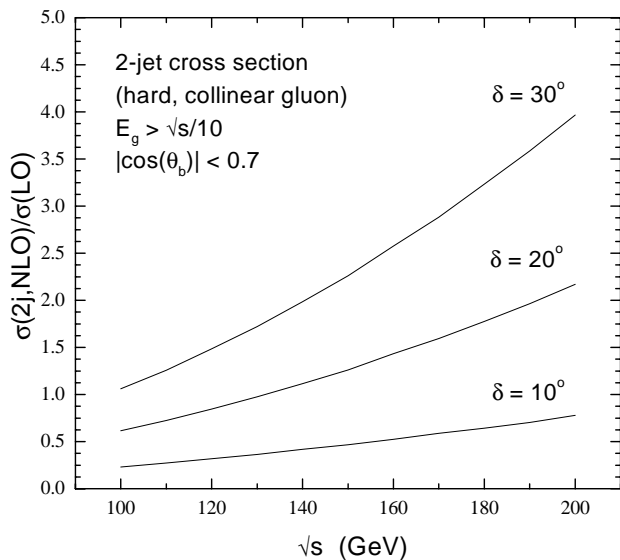
SV and  $\bar{I}$  contributions), the sums are indistinguishable. We can therefore conclude that for this part of the two-jet phase space (SV +  $\bar{I}$ ), the next-to-leading order cross section is between  $-2$  and  $-3$  times the leading order cross section.

Also shown in Fig. 3 is the ‘6 $\mathcal{F}$ ’ hard double logarithm contribution. Quite remarkably, this is close to the complete result, demonstrating that the net effect of the additional subleading real and virtual contributions is small and positive, at least for this choice of parameters (in particular  $\epsilon$ ). In fact to a very good approximation, in this energy range,

$$\sigma_{\text{SV}} + \sigma_{\bar{I}} = \sigma_{\text{LO}} \left[ 6\mathcal{F} + 0.3 + 0.001\sqrt{s} \text{ (GeV)} \right] \quad (12)$$

We may conclude that including the higher-order terms in the hard form factor *does* give an improved prediction for the cross section, and in particular restores positivity for this region of two-jet phase space.

The remaining part of the two- $b$ -jet cross section,  $\sigma_{\text{II}}$ , is shown relative to  $\sigma_{\text{LO}}$  in Fig. 4, for three choices of the cone size  $\delta$ . Consistent with the results presented in Fig. 3, the parameter  $\epsilon$  is again fixed at 0.1. Notice that this part of the two-jet cross section is large and positive



**Fig. 4.** Variations of the one-loop real bremsstrahlung contribution to the cross section. One can clearly see the removal of the  $\frac{m_q^2}{s}$  suppression as the available phase space is enlarged

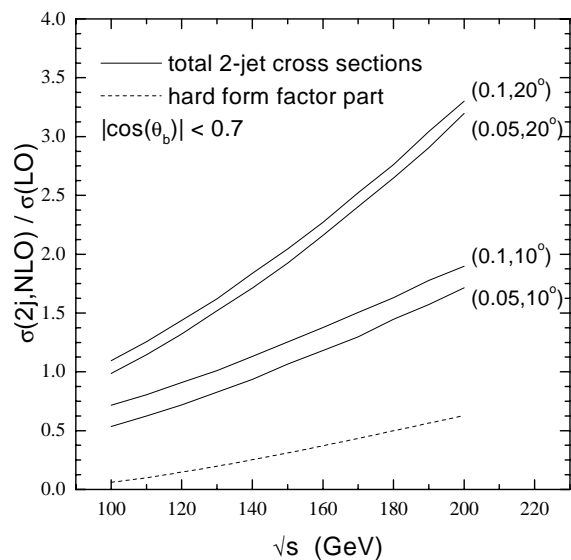
and approximately linear in  $\delta$ . The reason for this was first pointed out in [6]: for  $m_q \ll E_g \ll \sqrt{s}$  the  $q\bar{q}g$  cross section behaves as

$$\frac{d\sigma}{dE_g}(\gamma\gamma \rightarrow q\bar{q}g, J_z = 0) \sim \alpha^2 \alpha_s \frac{E_g^3}{s^3} [\dots] \quad (13)$$

i.e. with *no*  $m_q^2/s$  suppression. This part of the cross section is therefore dominated by hard gluon emission, i.e. the  $b$  quark in one of the two  $b$ -jets is likely to be soft. The non- $m_q^2/s$ -suppressed cross section has no collinear singularity either, and so the dependence on the cone size  $\delta$  is simply determined by phase space, i.e.  $\sigma_{\text{II}} \sim O(\delta)$ , for  $\delta \ll 1$ . An efficient suppression of the background to the Higgs signal will therefore necessitate selecting narrow  $b$  jets in which the heavy quark carries a large fraction of the jet momentum.

## 4 Summary and conclusions

In the previous section we have studied two types of correction to the ( $J_z = 0$ )  $\gamma\gamma \rightarrow q\bar{q}$  cross section: the all-orders resummed hard quark mass double logarithms, and the exact next-to-leading order (one-loop) corrections. Motivated by the topology of the Higgs signal, to which our contributions are a background, we have focused on the two- $b$ -jet cross section, defined here by two parameters, an ‘energy outside the cone’ parameter  $\epsilon$  and a cone size parameter  $\delta$ . Furthermore we have shown that the part of the NLO cross section corresponding to virtual and soft real gluon emission is dominated by the leading double hard logarithm. This emphasizes the importance of resumming these contributions. On the other hand, the



**Fig. 5.** The total two- $b$ -jet cross section (i.e. exact next-to-leading order contribution plus resummed hard form factor) normalized to leading order, for various jet parameters ( $\epsilon, \delta$ ). Also shown (dashed line) is the hard form factor part alone

hard collinear gluon part of the cross section is sizable and depends quite sensitively on the jet parameters.

Our results are summarized in Fig. 5, where we display (solid curves) the total two-jet cross section (i.e. exact next-to-leading order contribution plus resummed hard form factor) for four choices of the parameters ( $\epsilon, \delta$ ). Also shown (dashed curve) is the hard form factor part alone. This is of course independent of the jet definition. The form factor evidently gives a significant contribution to the total cross section, especially for narrow jets. From Fig. 1, we see also that using only the one-loop ( $6\mathcal{F}$ ) part of the hard form factor would result in a *negative* two-jet cross section for narrow jets, a result first pointed out in [7]. With the resummed form factor, positivity and stability is restored.

An additional potentially important higher-order contribution for large energies comes from the square of the *imaginary* part of the one-loop box diagram. An explicit calculation gives [7]

$$\begin{aligned} \frac{d\sigma_{\text{Im}}}{d\sigma_{\text{Born}}}(\gamma + \gamma \rightarrow q + \bar{q}, J_z = 0) & \\ & \approx \frac{\alpha_s^2}{18\pi^2} \frac{s}{m_b^2} \cos^2 \theta (1 - \cos^2 \theta) \\ & \leq \frac{\alpha_s^2}{72\pi^2} \frac{s}{m_b^2} \end{aligned} \quad (14)$$

Note that, unlike  $\sigma_{\text{Born}}$ , this contribution has no  $m_q^2/s$  suppression and therefore eventually dominates the cross section at very high scattering energy. For the energies considered here, however, it is negligible.

What can we say about the corrections not included in Fig. 5? Of course the unknown exact NNLO corrections

could be important, especially for the hard collinear contributions to the two-jet cross section. We note here that, as at NLO, there is no  $m_q^2/s$  suppression of such terms, but tightening the two-jet requirement will tend to reduce their significance. We might again expect that the hard form factor is the dominant part of the virtual/soft multigluon contributions. Note also that as long as the parameter  $\epsilon$  is not taken to be too small, in which case large logarithms ( $\sim \alpha_s \log(m_q^2/s) \log \epsilon$ ) and the effective scale of  $\alpha_s$  would require resummation, the Sudakov form factor (see (9)) should not play a major role. Finally, it is possible that the sub-leading (e.g.  $\alpha_s^n \log^{2n-1}(m_q^2/s)$ ) logarithmic contributions to the hard form factor are important (although this is not suggested by our exact next-to-leading order studies). The calculation beyond NLO would seem to be an extremely difficult task however [10].

In conclusion, we have demonstrated the numerical importance of the ‘hard’ (non-Sudakov)  $\log^2(m_q^2/s)$  contributions to the  $J_Z = 0$   $\gamma\gamma \rightarrow b\bar{b}$  cross section at typical photon collider energies relevant for intermediate-mass Higgs searches. Our study, although it includes for the first time all the currently available theoretical information, is of course far from complete.<sup>6</sup> We have not, for example, included running  $\alpha_s$ , hadronization or detector effects. These would require dedicated Monte Carlo studies, such as those performed in [6] for example. In this context, it is worth pointing out that including the new hard logarithm form factor in such studies should be straightforward.

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<sup>6</sup> A more detailed phenomenological analysis will be presented elsewhere [14].